BALANCE VELOCITY

AND

BASAL SHEAR STRESS

DETERMINATIONS

OF AN ICE STREAM

IN

WEST CENTRAL

GREENLAND

A Thesis

Presented in Partial Fulfillment
of the Requirements for the
Degree Bachelor of Science

By

Michael A. Kipp

Approved by

Ian Whillans
Advisor, Department of
Geology and Mineralogy

date 11/27/78

ACKNOWLEDGEMENTS

I would like to thank Doctors I. M. Whillans, C. Bull and G. Mckenzie, for their guidance and assistance. I would also like to thank Dave Balcer for his helpful comments and his expertise on trout fishing in the high altitude ice streams of Greenland.

CONTENTS

ABSTRACT

A theoretical model based on balance velocity and basal shear stress is applied to an ice stream flowing into Jakobshaven Isford glacier Greenland.

Balance velocity is defined by Whillans (1977) as a measure of the time change of the mass of a column through the ice thickness. Balance velocities were calculated for eleven stations along the flow band from surface data such as ice thickness, accumulation, and flow band width.

Basal shear stress is the stress located near the base which results in the slippage of the ice over the permafrost resulting in movement of the ice mass. Basal shear stress was calculated at the eleven stations using data such as ice thickness and surface slope.

The results from the calculations were then plotted on a graph to illustrate their relationship along the flow band. Increasing values of basal shear stress were found associated with increasing values for balance velocity. This is typical of glaciers which are frozen to their base. However, Weertman (1969) theoretically concludes that a water layer would exist at the base of the ice stream. The thickness of the layer which is unknown could then account for some of the variations in the velocities. If the water layer exists, the values for basal shear stress would seem too large to be representative of the area.

INTRODUCTION

The large expanse of the Greenland ice sheet covers some 721,000 mi^2 of the continent. Of this large area a comparatively small amount has been researched adequately enough to provide us with a solid understanding of ice movement. Due to this lack of field data, we must apply theoretical models of ice movement in order to initially understand these areas.

In this thesis a theoretical model based on balance velocity and basal shear stress is used to provide us with a simplified initial view of ice movement.

The term balance velocity refers to an equation derived by Whillans (1977) and is calculated largely from surface data such as accumulati~~ng~~on, ice thickness and flow line dimensions.

Whillans (1977) first applied balance velocity to the Byrd Station Strain Network in Antarctica where measured ice velocities were available. By comparing the calculated balance velocity to the measured velocity Whillans concluded that balance velocity describes 80% of the true mean velocity (WHILLANS, 1977).

The term basal shear stress refers to the stress occurring in the lowest layers of a moving ice mass. This stress results in the slippage of the ice over the permafrost resulting in ice movement. Although other stress factors such as hydrostatic and internal stress are present in moving ice, it is generally considered that basal shearing stress is the major stress factor occurring in moving ice (PAT ERSON, 1969). Basal shear stress is calculated from data such as surface slope and ice thickness.

Theoretical principles of perfect plasticity and steady state are also applied in this study. Perfect plasticity assumes that the ice flows plastically under its own weight as a response to basal shearing stress (NYE, 1951). Internal deformation within the ice is considered negligible and therefore the ice is flowing as a solid continuous sheet. Steady state implies that the ice mass is in equilibrium and that such factors as temperature, accumulation, and ice thickness along the ice sheet remain constant. These theoretical assumptions were necessary for the simplification of this study and will be discussed in detail under the heading THEORY. This model based on balance velocity and basal shear stress was then applied to an ice stream in west central Greenland.

The term ice stream refers to a zone of rapidly moving ice occupying depressions in the topography and bordered by crevasses (SUGDEN and JOHN, 1976, p. 64). This is synonomous with the term outlet glacier. These ice streams are numerous on the Greenland ice sheet and are found radiating outward from the north and south ice domes. The ice stream

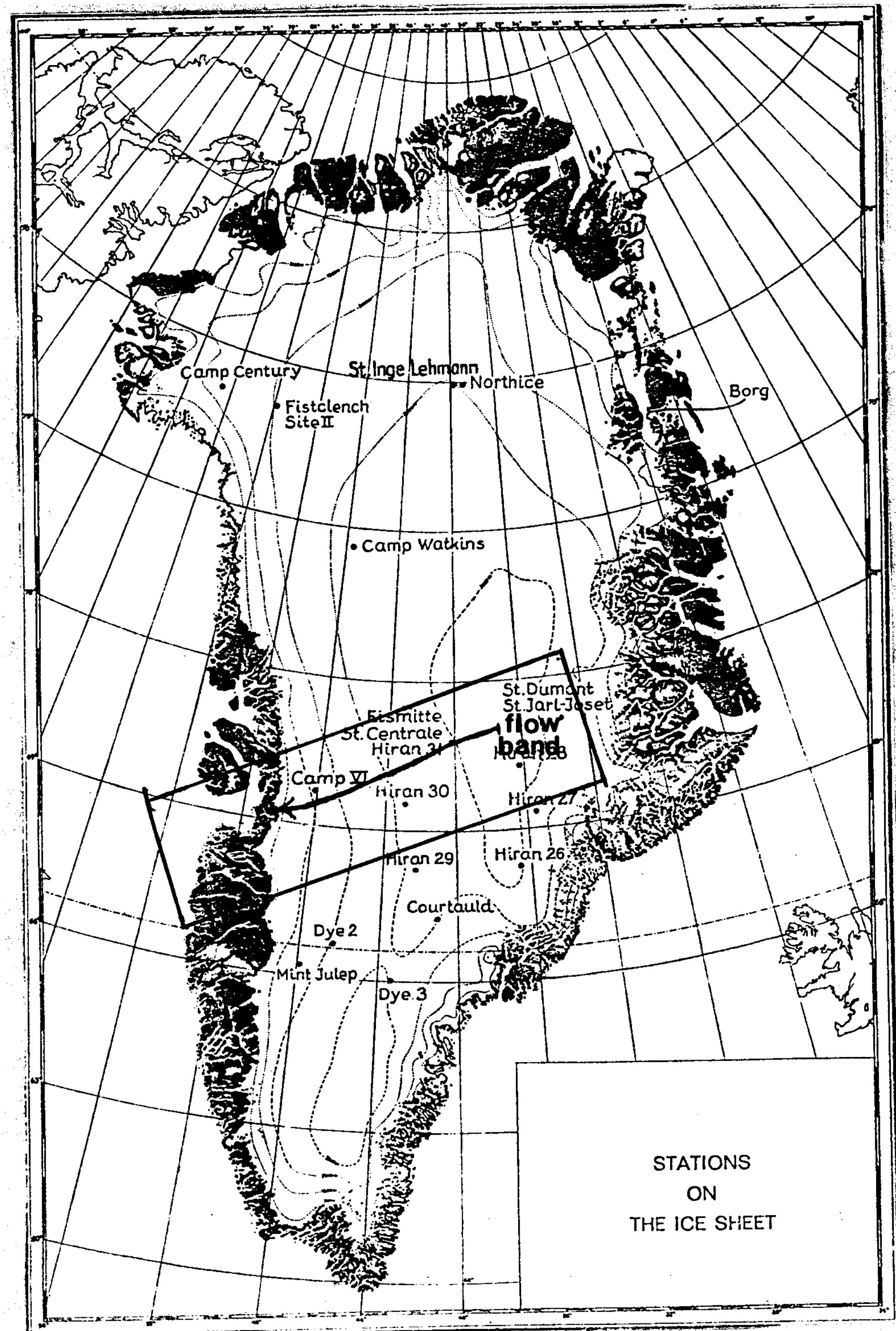


Fig. 1. MAP SHOWING LOCATION OF FLOW BAND

used in this study was established by the author using flow line techniques in order to describe typical ice flow in the area. Therefore this ice stream may deviate in dimensions from the ice stream existing in the area. The ice stream is located by its crest some 125 meters northeast of Station Centrale Eismitte and by its terminus located at the highly active Jakobshaven Isbral glacier (Fig. 1).

Application of this model helped to form a clearer understanding of the validity and the relationship between calculated balance velocity and basal shear stress. This application also allowed for a greater understanding of velocities along the ice stream and the factors controlling these velocities. Furthermore, this study provides an initial simplified view of ice movement in the area, which may be of importance to field studies in the future.

THEORY

As stated in the introduction, the theoretical assumptions of perfect plasticity and steady state are established in this study.

PERFECT PLASTICITY

Ice exhibiting perfect plasticity is considered to be an isotropic homogeneous solid flowing plastically under its own weight (NYE, 1951). The above statement suggests that no preferred orientation, or internal deformation occurs within the ice mass. Perfect plasticity further assumes that ice does not deform below a critical yield stress of one bar and when this critical value is reached the ice deforms indefinitely due to basal shear stress. Although a critical yield stress has never been established in laboratory experiments repeated studies show that most basal shear stress is between a minimum value of .5 bars and a maximum value of 1.5 bars (PATTERSON, 1969).

Application of perfect plasticity to an ice sheet was first completed by Nye in Greenland. Nye determined theoretically that the temperature rose rapidly near the base of the ice due to heat of friction and geothermal heat (FRISTRUMP, 1966). Since deformation is greater at higher temperatures, Nye concluded that the largest amount of movement would occur at the base due to basal shear stress and movement could be considered purely plastic.

STEADY STATE

The theoretical concept of steady state includes two assumptions. First it assumes that the net balance of the ice stream is equal to zero. This implies that the rate of accumulation or input is equal to the rate of ablation or output and therefore the ice stream is in equilibrium.

It further suggests that the ice thickness remains constant throughout the ice stream. The second assumption is that of constant temperature throughout the ice stream.

Steady state rarely occurs in nature but is assumed in this study due to the lack of data which would support a diminishing or increasing ice mass. Steady State is therefore a compromise between a diminishing increasing ice mass.

METHODS

Methods in this thesis involved the combination of up-to-date surface data from the vicinity under study. This surface data included surface topography, basal topography, isoheytal or accumulation contours of the area, and a flow band diagram defining the ice stream. By transferring these surface data onto separate transparencies it was then possible to combine the information. All of this information was instrumental in the calculation of balance velocity and basal shear stress.

SURFACE TOPOGRAPHY

Surface topography (PLATE 1) was taken from the map compiled by Weidick with assistance from Escher in 1975. The topography was developed by these men using methods of aerial photo interpretation and with reference to some earlier maps of Greenland.

The topographic contours are at (300-m) intervals. These contours were too broad for certain points along the flow band and therefore subintervals were determined. That is a point midway between the contours 2100 meters and 2400 meters would be considered as 2250 meters.

BASAL TOPOGRAPHY

Basal topography (PLATE 2) was also taken from the map compiled by Weidick and Escher (1975). This information was obtained by the use of modern seismic gravitational methods. The basal topography is at (250-m) intervals and the method used for determining subintervals was also applied to these contours.

ACCUMULATION DATA

The most recent isoheytal contours were found in Mock, [1967, JOURNAL OF GLACIOLOGY, VOL. 6, p. 795-803]. Researchers responsible for this information are: [as follows] Koch and Wegner, 1930; Languay, 1961; Lister, 1961; Benson, 1962; Rayle and Davis, 1962; and Mock and Weeks, 1966. The body of these data were determined using stratigraphic pit methods at various stations on the ice sheet. Mock and Weeks (1966) developed a statistical method of determination which produced further accumulation data. All of the individual studies were then combined by Mock to represent regional accumulations trends on the Greenland ice sheet (PLATE 3).

The isoheyts are measured in (gr cm^{-2} a^{-1}) of snow and are described in five gram intervals. For the purpose of calculation in this study, the units were converted to ($kg \cdot m^{-2} \cdot a^{-1}$).

FLOW BAND ESTABLISHMENT

The flow band pattern (PLATE 4) was established by the author in order to define a typical ice stream in the area. This flow band pattern consists of two flow lines which define the width of the ice stream from the surface to the base of the ice. This is in accordance with the theory presented by Whillans (1977) which concludes that flow lines are in the same direction at depth as they are at the surface. This is inferred by the fact that balance velocity calculated from surface flow line data closely resembles the true mean velocity throughout the ice stream.

It can be seen from (Plates 1 and 4) that the flow band consists of two flow lines which are established perpendicular to the surface contours. This perpendicular orientation then determines the variations in width along the flow band.

Along the length of the flow band distances in increments of 50 km are established and designated as stations one through eleven. Station one is located at the ice divide where velocities are theoretically assumed to equal zero and station eleven is located near the terminus. These eleven stations were then taken as reference points along the ice stream for the calculations of balance velocity and basal shear stress.

CALCULATIONS

For the purposes and scope of this study, the author has deleted the mathematics used to derive the equation for balance velocity. The derivation for balance velocity in its entirety can be found in Whillans (1977). The equation for basal shear stress was taken from (PATERSON, 1969, p. 90).

BALANCE VELOCITY

The term balance velocity refers to the velocity calculated from mass balance summated up glacier. This concept is based on the principle of mass balance which states that the mass of ice flowing through a vertical cross section is equal to the snow accumulation up glacier. This can be expressed in the form:

$$q w \rho z = \sum \rho \dot{b} w \Delta x \qquad [1]$$

MASS OF ICE AT VERTICAL CROSS SECTION = MASS OF ICE UP GLACIER

Where the terms are defined in Fig. 2.

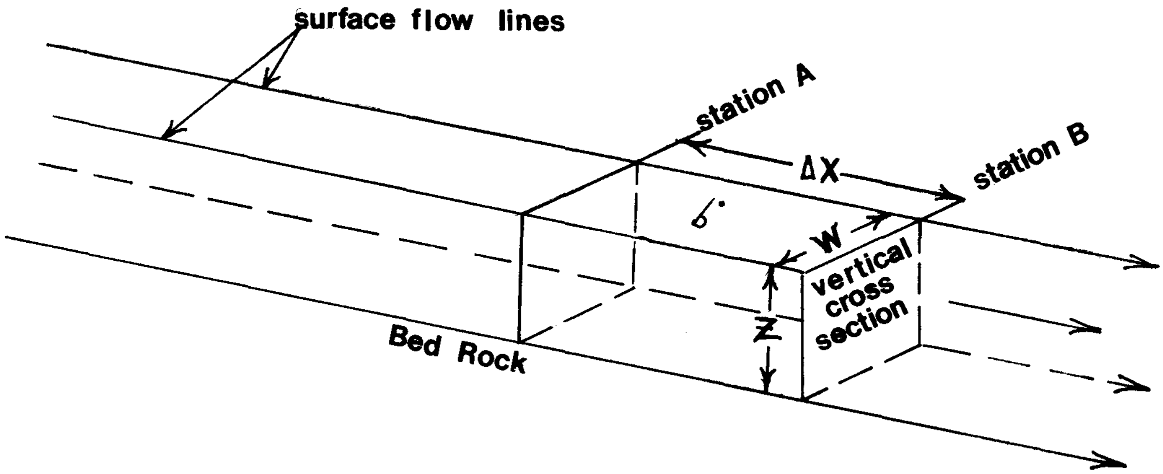


Fig. 2. SCHEMATIC DIAGRAM ILLUSTRATING THE TERMS IN EQUATION 1

Then by manipulating equation 1 we arrive at the equation for balance velocity.

$$q = \frac{1}{w \rho z} \sum w \dot{b} \Delta x \qquad (2)$$

In equation (2) q is balance velocity measured in $(m \cdot a^{-1})$ for each station along the flow band. This balance velocity increases from zero at the crest to $430\ m \cdot a^{-1}$ at station eleven.

In the expression $\frac{1}{w \rho z}$, W describes the flow line width measured at each station in meters, P is the mean density of ice or 912 kg M^{-3} and Z is the ice thickness at each station measured in meters. The ice thickness is determined by taking the difference between surface and basal elevations. The second portion of equation (2) in the form $\sum w \dot{b} \Delta x$ represents the summation of the mass of snow accumulation up glacier. In this expression W represents the average flow line width between stations, $(\dot{b})$ represents the average snow accumulation between stations and the term (ΔX) is the distance between stations measured in meters. For a table of balance velocities, refer to (TABLE 1).

BASAL SHEAR STRESS

Basal shear stress is the stress located near the base of the ice mass which balances the down slope component of the weight of the ice mass. The values for basal shear stress are calculated using equation (3).

$$\tau = \rho g z \sin \alpha \qquad [3]$$

TABLE I [BALANCE VELOCITY AND DATA USED IN CALCULATIONS]

STATION	SURFACE ELEVATION (M)	BASAL ELEVATION (M)	Z ICE THICKNESS (M)	X DISTANCE ALONG FLOW LINE (M)	ΔX DISTANCE BETWEEN STATIONS (M)	W WIDTH OF FLOW BAND (M)	$\dot{b}$ ACUMULATION ($Kg\,M^{-2}a^{-1}$)	$W\dot{b}\Delta X$ ($Kg\,a^{-1}$)	$\Sigma W\dot{b}\Delta X$ ($Kg\,a^{-1}$)	q BALANCE VELOCITY ($m\,a^{-1}$)
1				0		50,000	275			0
					50,000	49,000	275	6.7375×10^{11}		
2	3150	-125	3275	50,000		48,000	275		6.7375×10^{11}	4.699
					50,000	47,000	300	7.05×10^{11}		
3	3000	-125	3125	100,000		46,000	325		1.3787×10^{12}	10.516
					50,000	42,500	350	7.4375×10^{11}		
4	2850	125	2725	150,000		39,000	375		2.1224×10^{12}	21.898
					50,000	39,500	412.5	8.1468×10^{11}		
5	2850	125	2725	200,000		40,000	450		2.9370×10^{12}	29.545
					50,000	39,500	487.5	9.6281×10^{11}		
6	2550	125	2425	250,000		39,000	525		3.8998×10^{12}	45.213
					50,000	38,000	525	9.975×10^{11}		
7	2400	-125	2375	300,000		37,000	525		4.8973×10^{12}	61.107
					50,000	35,500	525	9.3187×10^{11}		
8	2100	-125	2225	350,000		34,000	525		5.8219×10^{12}	84.489
					50,000	35,500	500	8.875×10^{11}		
9	1800	0	1800	400,000		37,000	475		6.7166×10^{12}	110.58
					50,000	34,500	450	7.7625×10^{11}		
10	1500	-125	1625	450,000		32,000	425		7.4928×10^{12}	157.99
					50,000	26,000	425	5.525×10^{11}		
11	900	-125	1025	500,000		20,000	425		8.0453×10^{12}	430.32

Where the terms are defined in Fig. 3.

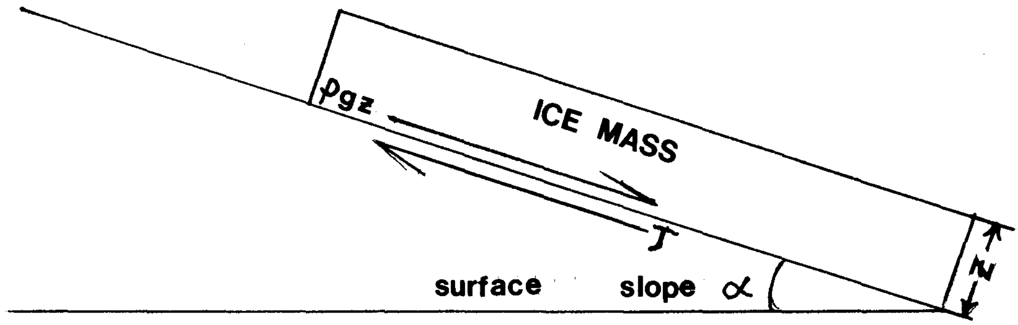


Fig. 3. SCEMATIC DIAGRAM ILLUSTRATING THE TERMS IN EQUATION 2 WHERE ICE IS CONSIDERED A PRALLEL - SIDED SLAB

In equation (3) the expression $\rho g z$ describes the component of weight of the ice parallel to the basal surface. The term ρ is the density of the ice and g is the constant of gravity. Both g and ρ remain constant and therefore the basal shear stress measured in bars depends upon values of ice thickness Z and surface slope α . High values of surface slope and ice thickness therefore result in high values for basal shear. For a table of basal shear stress data, refer to (TABLE 2).

TABLE II [BASAL SHEAR STRESS AND DATA USED IN CALCULATION]

STATION	X DISTANCE ALONG FLOW LINE	ΔE CHANGE IN ELEVATION	ΔX HORIZONTAL DISTANCE	α SURFACE SLOPE	SIN α	Z ICE THICKNESS	τ BASAL SHEAR STRESS
	(M)	(M)	(M)	DEGREES		(M)	BARS
1	0		0				
2	50,000	150	100,000	$8.59 \cdot 10^{-2}$	$1.4992 \cdot 10^{-3}$	3275	$4.3882 \cdot 10^{-1}$
3	100,000	300	100,000	$1.718 \cdot 10^{-1}$	$2.998 \cdot 10^{-3}$	3125	.83734
4	150,000	150	100,000	$8.59 \cdot 10^{-2}$	$1.4992 \cdot 10^{-3}$	2725	$3.6512 \cdot 10^{-1}$
5	200,000	300	100,000	$1.718 \cdot 10^{-1}$	$2.998 \cdot 10^{-3}$	2725	.7301617
6	250,000	450	100,000	$2.57 \cdot 10^{-1}$	$4.4854 \cdot 10^{-3}$	2425	$9.72 \cdot 10^{-1}$
7	300,000	450	100,000	$2.57 \cdot 10^{-1}$	$4.4854 \cdot 10^{-3}$	2375	$9.521 \cdot 10^{-1}$
8	350,000	600	100,000	$3.437 \cdot 10^{-1}$	$5.9986 \cdot 10^{-3}$	2225	1.19289
9	400,000	600	100,000	$3.437 \cdot 10^{-1}$	$5.9986 \cdot 10^{-3}$	1800	$9.65 \cdot 10^{-1}$
10	450,000	900	100,000	$5.156 \cdot 10^{-1}$	$8.9987 \cdot 10^{-3}$	1625	1.3069
11	500,000	600	50,000	$6.8 \cdot 10^{-1}$	$1.1867 \cdot 10^{-2}$	1025	1.0871

ACCURACY OF DATA

Certain aspects of the body of data presented in this thesis should be kept in mind when considering the results of the balance velocity and basal shear stress calculations.

For example, the basal elevations were determined to have an accuracy of ± 125 m and the surface elevation an accuracy of ± 150 m. This accuracy was determined by modifying the measured elevations by plus or minus the accuracy factors stated above. The resultant values for balance velocity varied from the values in (TABLE 1) by no more than 14%.

These correction factors were also applied to the ice thicknesses used in the calculation of basal shear stress. The resultant values deviated from the values in (TABLE 2) by a maximum of 15%. However, the surface slope was determined to be the most critical factor involved in basal shear stress calculations. Surface slope is determined by the change in surface elevation over a measured distance along the flow line. Therefore by modifying the change in elevation by the maximum variance of ± 150 m the values for surface slope and basal shear stress were found to deviate from values in (TABLE 2) by 50%.

The accumulation data was determined to have an accuracy of $\pm 5 \text{ gr cm}^{-2} \text{ a}^{-1}$ for the purposes of balance velocity calculations.

Nevertheless possibilities of error in stratigraphic accumulation studies are rather high and resulting isoheytal contours must be viewed as regional trends and not exact predictions (MOCK, 1967).

Error in measurement of flow line width and distances along the flow line were found to be negligible to the outcome of the calculations and are considered accurate within the large scale used in this study.

In considering all of the possibilities of error in surface data, I would have to conclude that both balance velocity and basal shear stress calculations have an accuracy of around 50%.

DISCUSSION and CONCLUSION

For the purposes of illustrating the relationship between balance velocity and basal shear stress, the results from Tables 1 and 2 were plotted on a graph (Fig. 4).

The values for balance velocity as seen in (Fig. 4) show an increase from a theoretical value of zero at station one to a maximum of 430 $m \cdot a^{-1}$ at station eleven. This increasing trend in velocity from crest to terminus is typically observed on ice streams similar to the one under study. For example, the ice stream originates at high altitudes where due to its remoteness from water, accumulation is small. As the ice stream flows to lower altitudes, the mass balance

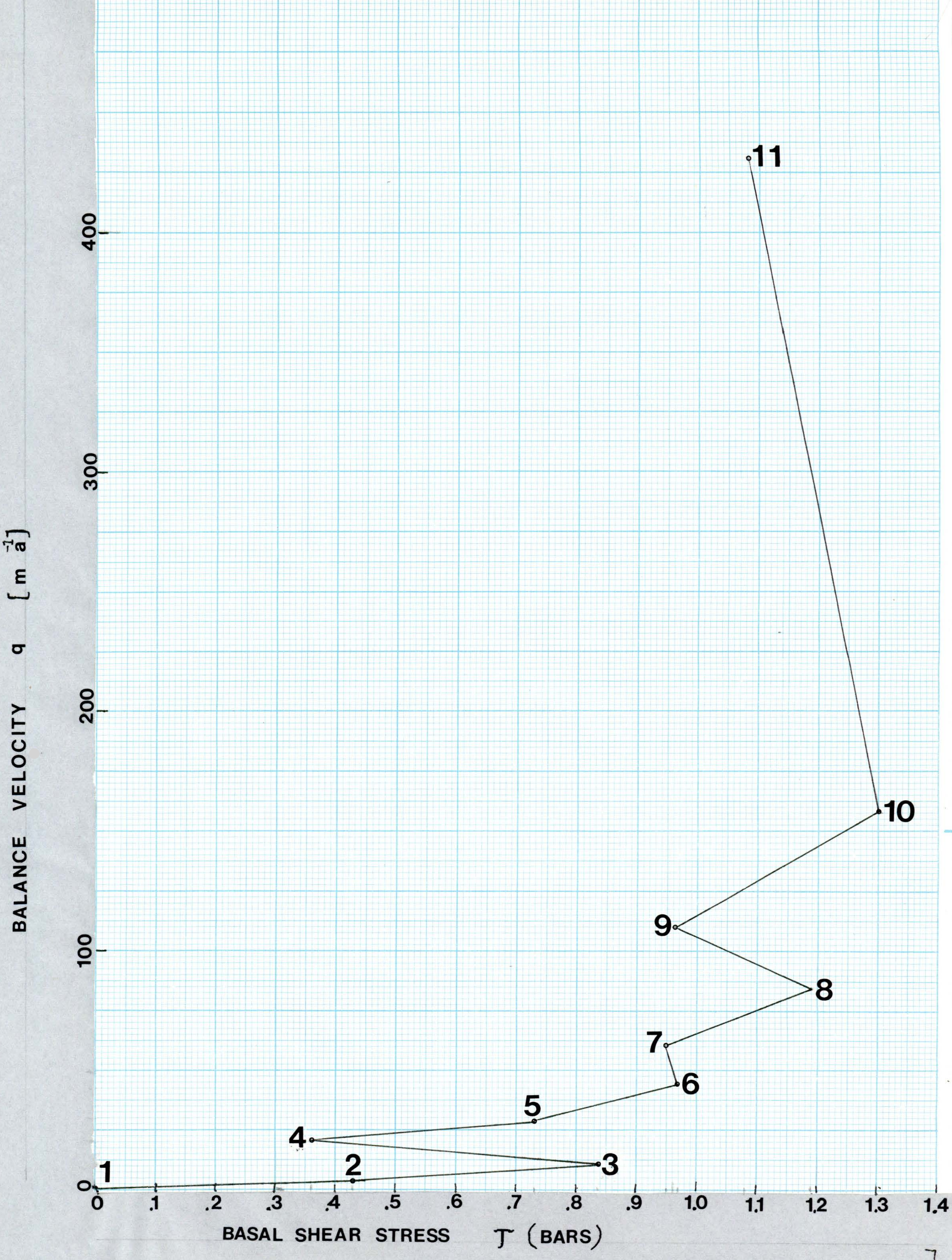


Fig [4] [GRAPHICAL REPRESENTATION OF BALANCE VELOCITY AND BASAL SHEAR STRESS] or accumulation increases. This can be seen on (TABLE 1) where accumulation $\dot{b}$ shows an increasing trend down glacier with a slight tapering off towards the terminus. The equilibrium line which is a generalized boundry between zones of accumulation and ablation is located near the terminus of the ice stream where major ablation is due to calving into the sea. This equilibrium line marks the zone of maximum velocity (PAT ERSON, 1969).

Bauer in 1960 also calculated the velocities up glacier from Jakobshaven glacier (FRISTRU P, 1966). At 90 Km up glacier or roughly equivalent to station nine, Bauer recorded a velocity of 118 $m \cdot a^{-1}$. The balance velocity at station nine (TABLE 1) is 110 $m \cdot a^{-1}$ or roughly 90% of Bauer's calculation. Bauer further determined the magnitude of the velocity at Jakobshaven glacier which is 20 to 30 $m \cdot day$ or 7 to 12 $Km \cdot a^{-1}$. It would therefore seem probable that large velocity increases would be observed near the vicinity of Jakobshaven glacier. For example in Fig. (4) velocities show a large increase by a factor of 2.72 from station ten to station eleven. This increase is no doubt partially due to an increasing slope of bedrock topography and the increasing convergence of the flow lines approaching the Jakobshaven glacier.

As seen in (TABLE 1) and PLATE 2) the basal elevations along the ice stream vary from +125 to -125 M. However, it is believed that these basal elevations are not representative of stations ten to eleven where large velocity increases would be associated with a much steeper basal slope. Up glacier from station ten, the basal topography is more representative as can be seen by the gradual increasing nature of the velocities (TABLE 1). As seen in (Fig. 4) the velocities increase by an average factor of 1.37 from station four to station ten. This velocity increase can be contributed to the gradual slope of the underlying bedrock, the gradual convergence of the flow band, and the increased mass balance down the ice stream.

The basal shear stresses seen in (Fig. 4) fluctuates from a minimum value of .36 bars at station four to a maximum value of 1.3 bars at station ten. Since it is assumed that basal shear stress is the movement mechanism along the ice stream, it would theoretically be expected to increase in value as velocity increases. The anomalous nature of the values are therefore contributed to the inaccuracy of the data used in the calculations. As stated previously, the values for basal shear stress are determined to have an accuracy of approximately 50%. Therefore, by eliminating the fluctuations due to error, a pattern of increasing values from station one to station eleven emerges. As seen in (TABLE 2) the increasing values of surface slope are the major determinants of the increasing trend in basal shear stress.

The behavior of velocities and basal shear stresses of the ice stream are possibly related to temperatures at the base of the ice. If the base of the ice is at pressure melting temperature, a component of movement termed basal sliding is introduced. Basal sliding refers to the movement of the ice over the bedrock due to the lubricating tendencies of a small layer of water at the base.

Weertman (1964) concludes that a layer of water only a few millimeters thick would increase the sliding velocity by 40 to 100%. This increase in velocity is explained by the fact that the water layer would lift the ice mass and therefore decrease the amount of friction at the ice bedrock interface.

Weertman (1969) determined theoretically the basal temperatures due to heat of friction and geothermal heat. Weertman concluded that a large outlet glacier flowing into Jakobshaven Isford would have a water layer at its base from crest to terminus. If this condition exists, the component of basal sliding would have an important effect on ice velocities and basal shear stresses.

The thickness of the basal water layer is an important factor in considering the effects of basal sliding. As thickness of the water layer increases, the amount of friction due to irregularities and roughness of the bed decreases. Variations in the thickness of the water layer could therefore be correlated with velocity variations. As seen in Fig. 4, balance velocities increase by an average factor of 2 from stations two to four. This increase is by a factor of 1.37

from stations four to ten and by a factor of 2.72 from stations ten to eleven. All variables inherent to the calculations considered this pattern of velocity variation may be partly due to variations in water thickness at the base. Greater basal melting near stations two through four would theoretically increase basal sliding and possibly account for the two-fold velocity increases between the stations.

Under the conditions of pressure melting the basal shear stresses necessary to maintain a given velocity would decrease due to the lubricating tendencies of the water. Bull (1957) concludes that the low values of basal shear stress and high rates of movement measured in northeast Greenland indicate pressure melting. This is contrasted by the findings in northwest Greenland where basal shear stress is high, rates of movement are low, and the ice is assumed to be frozen to the base.

Repeated studies indicate that most basal shear stress varies from a minimum of .5 bars to a maximum of 1.5 bars (SUGDEN and JOHN, 1969, p. 27). Values for basal shear stress (TABLE 2) are fairly high if compared with the parameters stated above. These large values would normally be associated with the frozen base condition where basal shear stress is the major movement mechanism.

Due to the limitations of time and data, the temperatures at the base of the ice stream were not calculated. If basal sliding due to water lubrication does exist along the ice stream, the calculated values for basal shear stress may not be representative. It would, therefore, be helpful in future investigations to make these temperature determinations. The relationship of increasing basal shear stress with increasing balance velocity would describe the relationship expected in a cold based glacier. However, the large velocity increases, especially near the terminus, seem to indicate more than a response to basal shear stress alone.

REFERENCES

BULL, C. 1957. "Observations in North Greenland Relating to Theories of the Properties of Ice." Journal of Glaciology, Vol. 3, No. 21, March 1957. P. 67-72.

FRISTRU P, B. 1966. "The Greenland Ice Cap." University of Washington Press. P. 272, 235.

MOCK, S. J. 1967. "Calculated Patterns of Accumulation on the Greenland Ice Sheet." Journal of Glaciology, Vol. 6, No. 48, 1967. P. 795-803.

NYE, J. F. 1951. "The Flow of Glaciers and Ice-Sheets as a Problem in Plasticity." Proceedings of the Royal Society, Ser. H, Vol. 207, No. 1091. P. 554-72.

PAT ERSON, W. S. B. 1969. The Physics of Glaciers. Pergamon Press. P. 90, 64, 89.

SUGDEN, P. E. and B. S. John 1976. Glaciers and Landscape A Geomorphological Approach. Edward Arnold (Publishers) Ltd. P. 27, 64.

WHILLANS, I. M. 1977. "The Equation of Contimcicty and its Application to the Ice Sheet Near Byrd Station Antarctica." Journal of Glaciology, Vol. 18, No. 80, 1977. P. 359-370.

WEERTMAN, J. 1964. "The Theory of Glacier Sliding." Journal of Glaciology, Vol. 5. P. 287-303.

WEETMAN, J. 1969. "Water Lubrication Mechanism of Glacier Surges." Canadian Journal of Earth Sciences, Vol. 6, August. P. 932-933.

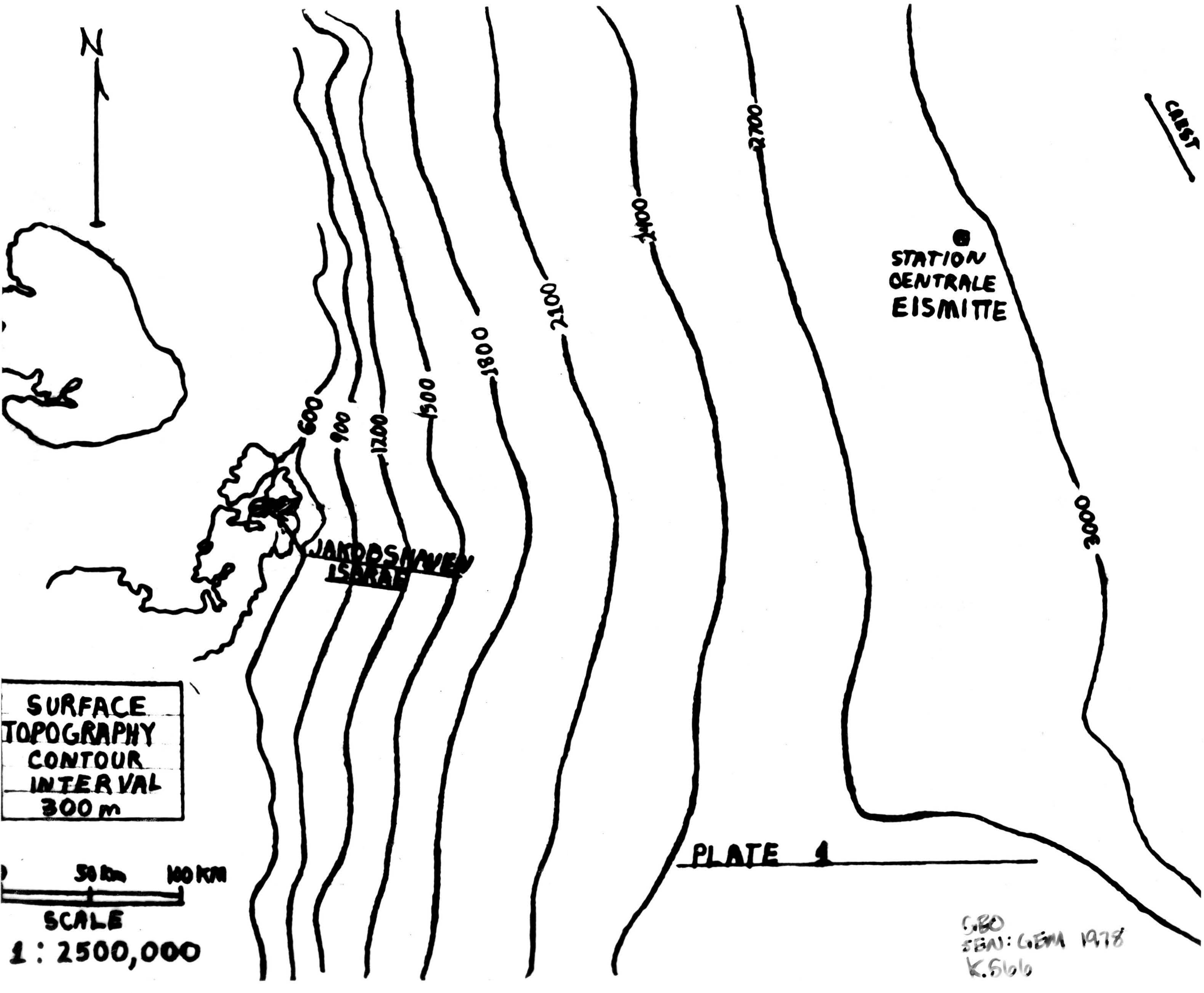

N
SURFACE TOPOGRAPHY CONTOUR INTERVAL 300 m
0
50 km
100 KM
SCALE
1 : 2500,000
JAKOBSHAVEN ISBRAE
600
900
1200
1500
1800
2100
2400
2700
3000
STATION CENTRALE EISMITTE
CREST

PLATE 1

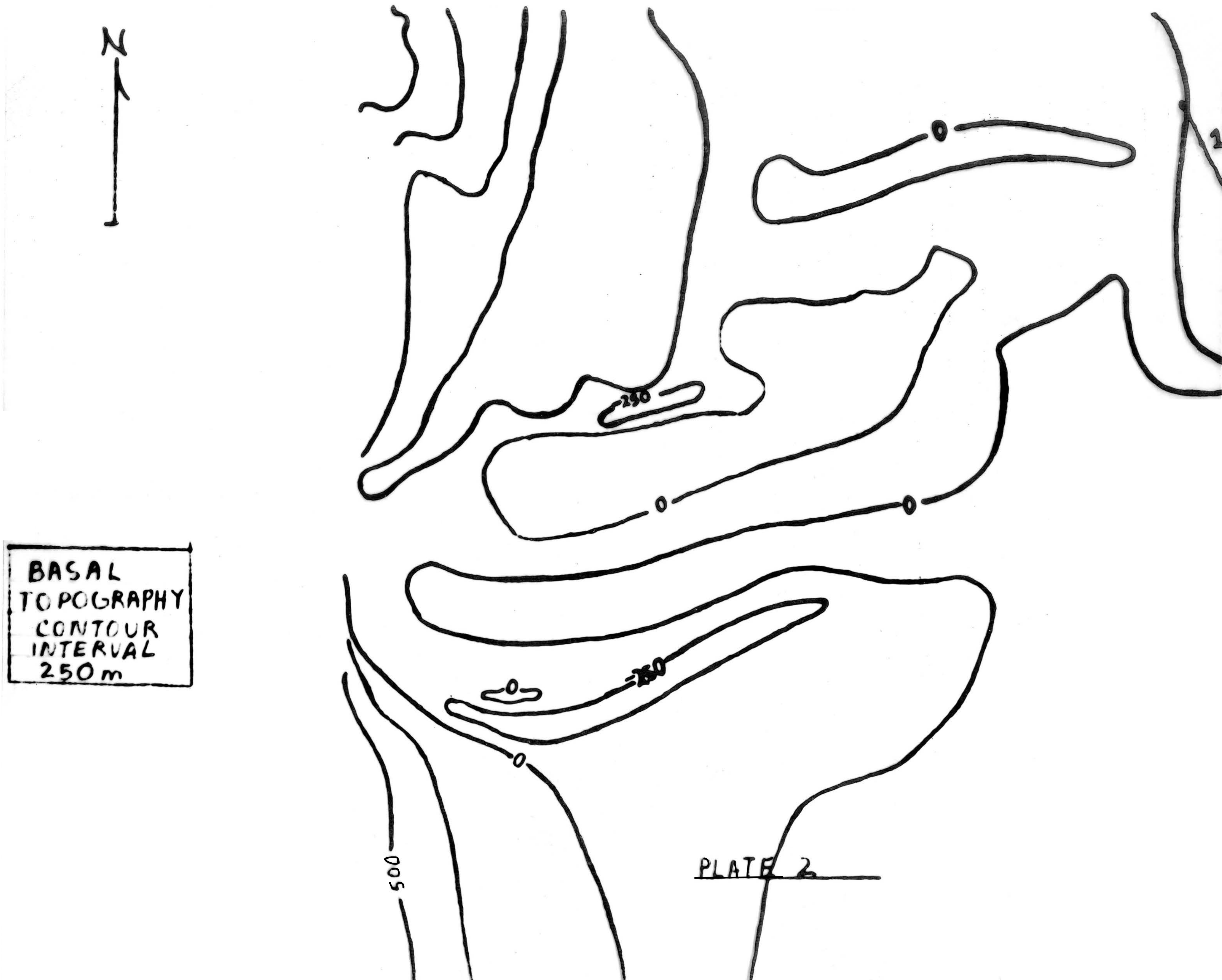
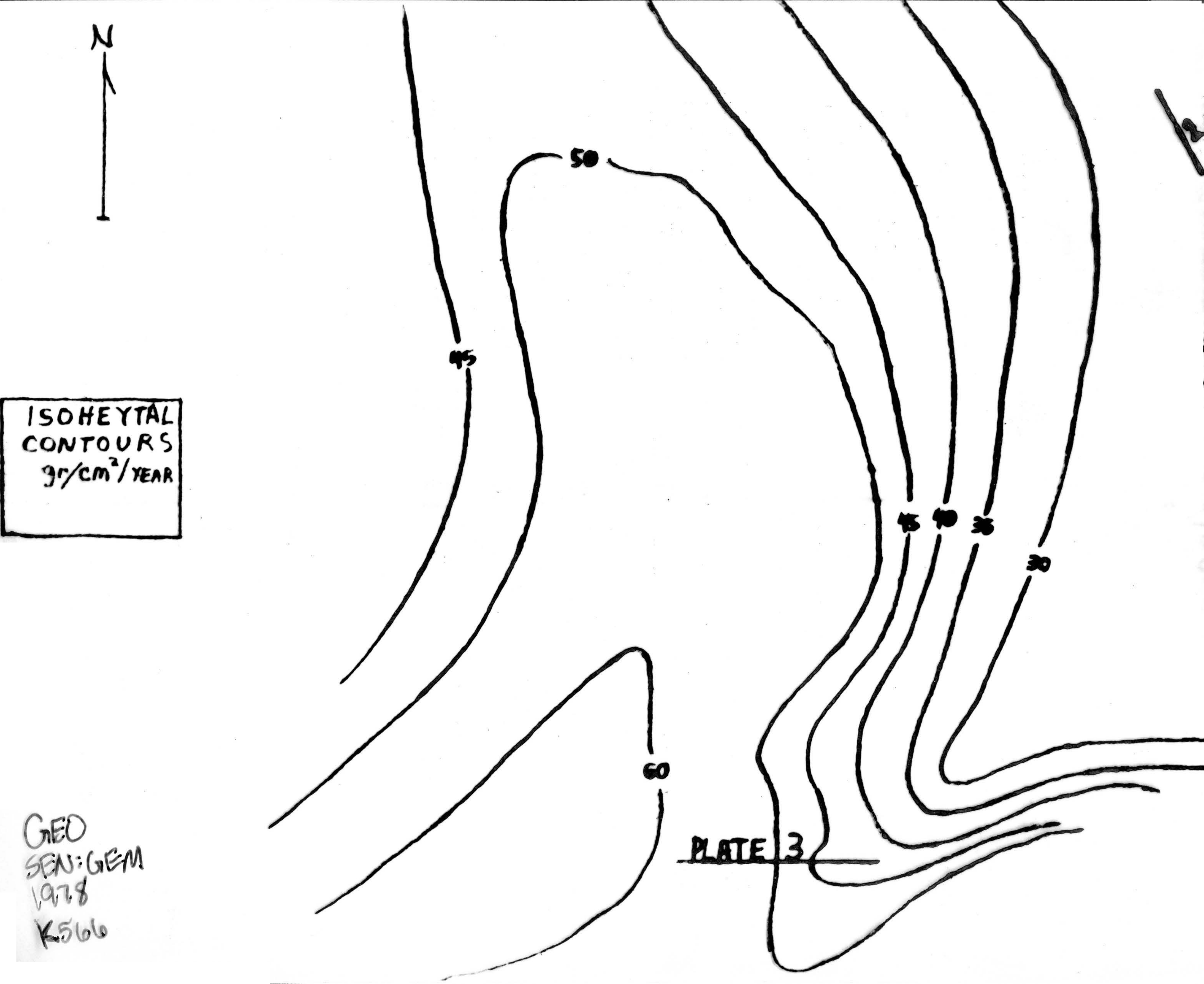

N
BASAL
TOPOGRAPHY
CONTOUR
INTERVAL
250 m
500
0
-250
0
0
0
0
0
-250

PLATE 2


N
ISOHEYTAL CONTOURS gr/cm²/YEAR
50
45
45
40
35
30
60
PLATE 3

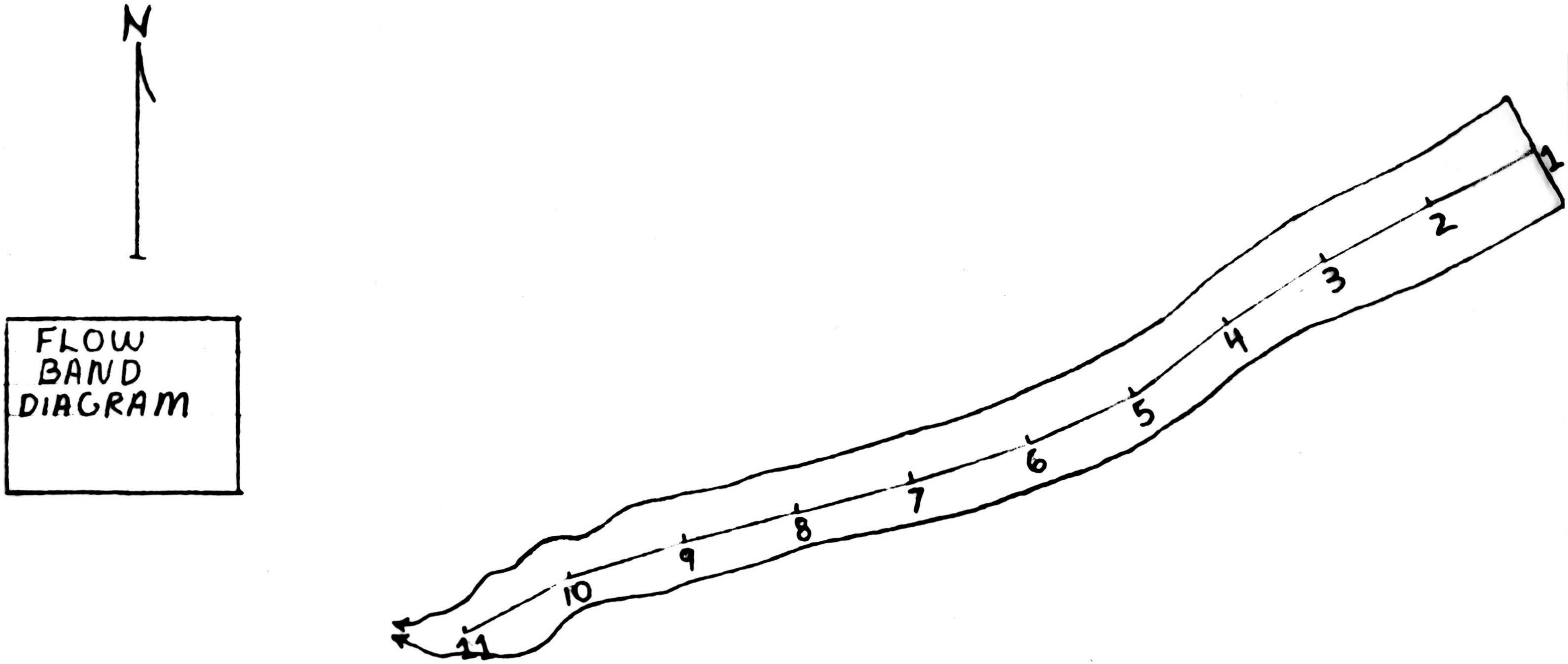


PLATE 4